Microcausality and tunneling times in relativistic quantum field theory

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We show, in the framework of a space-time resolved relativistic quantum field theory approach to tunneling, that microcausality precludes superluminal tunneling dynamics. More specifically in this work dealing with Dirac and Klein-Gordon fields, we first prove that microcausality holds for such fields in the presence of a background potential. We then use this result to show that an intervention performed on a localized region of an initial wave packet subsequently scattering on a potential barrier does not result in any effect outside the light cone emanating from that region. We illustrate these results with numerical computations for Dirac fermions and Klein-Gordon bosons.

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I. INTRODUCTION

While tunneling is ubiquitous in about any field described by quantum theory, the mechanism accounting for quantum tunneling has remained controversial [1]. In particular the issue of the time it takes a "particle" to tunnel across a potential barrier is not well-defined within quantum theory, given that on the one hand a quantum system is not a classical particle localized on a well-defined worldline, and on the other hand there is no time operator in quantum mechanics, and hence no time eigenvalues that would provide an unambiguous answer. Several theoretical approaches have been developed to capture the traversal time [2], many of which predict the possibility of superluminal times, including in first quantized relativistic contexts [3–11]. On the other hand works addressing the issue of tunneling times within a relativistic quantum field theory (QFT) framework are scarce [12].

In a recent work [13], it was shown by employing a space-time resolved formulation (see [14,15] and [16–20] for related work) of relativistic QFT that the tunneling dynamics does not exhibit any superluminal effects. That was done for the case of a Dirac field describing an electron wave packet. However, in that work the proof of causality relied on having an initial wave packet defined on a compact support and launched towards a potential barrier; causality was then addressed by modifying the initial wave packet and proving that such a change had no effect outside the light cone emanating from the compact support, including on any density transmitted through the potential.

It is well-known, however, that a single-particle QFT state cannot be defined on a compact spatial support as it must have infinite tails. And working with a wave packet defined over a compact support prompted us to employ in our previous work [13] a nonstandard form of QFT (introduced in [21]) based on a number symmetry rather than the fundamentally correct charge symmetry; in particular the field operators introduced in Ref. [21] and used to prove causality of the tunneling process in Ref. [13] are not the usual ones.

The aim of the present work is to give a proof of the causality of the tunneling process for standard QFT states and field operators. The protocol is different from the one elaborated for compact states, as rather than relying on the light cone emanating form a well-defined spatial support, we will be led to introduce an intervention modifying a wave packet (intrinsically defined with infinite tails) over a compact spatial support. We moreover extend the formulation of the proof so as to cover a spin-0 bosonic field in addition to the fermionic Dirac field. To this end, we will briefly recall in Sec. II the space-time resolved QFT formalism we will employ and how a wave packet is propagated. In Sec. III, we will introduce a protocol that will be used to prove that wave packet tunneling dynamics respects causality. We will first show that microcausality holds in the presence of a background field. We will then use this result to prove that if two initial wave packets differ only within a region $\mathcal{D}$ of their spatial density, then the density in a region causally disconnected from $\mathcal{D}$ on the other side of the potential barrier is identical for both wave packets. The proof will be illustrated by working out numerical computations in Sec. IV. We will close with a Discussion and Conclusion section.

II. SPACE-TIME RESOLVED QFT WITH A BACKGROUND POTENTIAL

A. Field operator

Our approach is based on a computational QFT framework [14], recently extended to treat particle scattering across a finite barrier [15] (see also [16–20] for related recent work). The formalism is essentially the same for Klein-Gordon (KG) or Dirac fields. The field operator takes the usual form,

$$\hat{\Phi}(t,x) = \int dp(\hat{b}_p(t)v_p(x) + \hat{d}_p^\dagger(t)w_p(x)), \qquad (1)$$

where $v_p(x)$ and $w_p(x)$ are respectively the positive and negative solutions of the first quantized free Dirac or KG equation[1]; hence v_p and w_p obey

$$i\hbar\partial_t v_p = H_0 v_p, \qquad (2)$$

where H_0 is the Dirac or Klein-Gordon Hamiltonian (in the latter case given in the so-called Feschbach-Villars form). We will consider in this work only one spatial dimension (hence, H_0 is two-dimensional both in the Dirac case, since spin-flip does not occur, and in the KG case). The creation and annihilation operators obey the commutation relations,

$$[\hat{b}_p^\dagger, \hat{b}_{p'}]_\epsilon = [\hat{d}_p^\dagger, \hat{d}_{p'}]_\epsilon = \delta(p-p'),$$
$$[\hat{b}_p^\dagger, \hat{d}_{p'}]_\epsilon = [\hat{d}_p^\dagger, \hat{b}_{p'}]_\epsilon = 0, \qquad (3)$$

where $\epsilon = 1$ for fermions and $\epsilon = -1$ for bosons; $\|0\rangle\rangle$ defines the vacuum state, i.e., $b_p\|0\rangle\rangle = d_p\|0\rangle\rangle = 0$.

The full first quantized Hamiltonian is

$$H = H_0 + V(x), \qquad (4)$$

where $V(x)$ is a rectangularlike potential barrier. The Hamiltonian H generates a unitary (or pseudounitary in the KG case) evolution. The QFT Hamiltonian density is given as usual by

$$\hat{\mathcal{H}} = \hat{\Phi}^\dagger(t,x)H\hat{\Phi}(t,x). \qquad (5)$$

As is well-known, it can be shown that the Heisenberg equation in the Dirac or KG cases becomes

$$i\partial_t\hat{\Phi}(t,x) = [\hat{\Phi}(t,x), \hat{\mathcal{H}}] = H\hat{\Phi}(t,x), \qquad (6)$$

so that the time evolution of the field operator is obtained from the first quantized unitary $U(t,t_0) \equiv e^{-i\hat{H}(t-t_0)}$ as

$$\hat{\Phi}(t,x) = e^{-i\hat{H}t}\Phi(0,x) = U(t)\hat{\Phi}(0,x), \qquad (7)$$

where the operator $\hat{H}$ is applied to $w_p(x)$ and $v_p(x)$. Note that U accounts for the evolution due to the background field. Comparing the time-evolved field operator to Eq. (1), one obtains the expressions of the time-evolved creation and annihilation operators,

$$\hat{b}_p(t) = \int dp'(U_{v_p v_{p'}}(t)b_{p'} + U_{v_p w_{p'}}(t)d_{p'}^\dagger),$$
$$\hat{d}_p^\dagger(t) = \int dp'(U_{w_p v_{p'}}(t)b_{p'} + U_{w_p w_{p'}}(t)d_{p'}^\dagger), \qquad (8)$$

where $U_{w_p v_{p'}}(t) = \int dxvw_p^\dagger(x)\sigma U(t)v_{p'}(x)$; here σ denotes the identity matrix in the case of fermions and the σ_3 Pauli matrix in the case of bosons.

B. Densities and wave packets

The density operators for positive and negative energy states are given by the usual expressions,

$$\hat{\rho}_+(t) = \iint dpdp'\hat{b}_p^\dagger(t)\hat{b}_{p'}(t)v_p^\dagger\sigma v_{p'},$$
$$\hat{\rho}_-(t) = \epsilon\iint dpdp'\hat{d}_p^\dagger(t)\hat{d}_{p'}(t)w_{p'}^\dagger\sigma w_p, \qquad (9)$$

where for fermions $\sigma = I$ (identity matrix) and $\epsilon = 1$ while for KG bosons $\sigma = \sigma_3$ is a Pauli matrix) and $\epsilon = -1$. The charge density operator is given by

$$\hat{\rho}(t,x) = \hat{\Phi}^\dagger(t,x)\sigma\hat{\Phi}(t,x), \qquad (10)$$

which can be rewritten as

$$\hat{\rho}(t,x) = \iint dpdp'\hat{b}_p^\dagger(t)\hat{b}_{p'}(t)v_p^\dagger(x)\sigma v_{p'}(x)$$
$$- \iint dpdp'\hat{d}_{p'}(t)\hat{d}_p(t)w_p^\dagger(x)\sigma w_{p'}(x)$$
$$= \hat{\rho}_+(t,x) - \hat{\rho}_-(t,x), \qquad (11)$$

where we used the anticommuting (commuting) property of the antiparticle creation and annihilation operators in the case of fermions (bosons).

In vacuum the space-time resolved density is the expectation value $\rho_0(t,x) = \langle\langle 0\|\hat{\rho}(t,x)\|0\rangle\rangle$. However we are interested in wave packet tunneling whereby a particle is initially prepared in a state $\|\chi\rangle\rangle$. This is a single particle state (here an electron, or a scalar boson) that can be generically written in case the particle has positive charge as

$$\|\chi\rangle\rangle = \int dpg_+(p;x_0,p_0)b_p^\dagger(0)\|0\rangle\rangle, \qquad (12)$$

[1] This choice is justified by the fact that the density of the tunneled wave packet is evaluated in field-free region. Other options are chosen in other contexts, like, for example, when the chiral charges for a Dirac field are the relevant observable, the basis diagonalizing the chiral charge operator is chosen [22].

where $g_+(p)$ are the wave packet amplitudes in momentum space and x_0 (p_0) denotes the initial average position (momentum). The time-dependent density in the presence of a wave packet is given by

$$\rho(t,x) = \langle\langle \chi \| \hat{\rho}(t,x) \| \chi \rangle\rangle. \tag{13}$$

$\rho(t,x)$ accounts for the full charge density; the part due to the wave packet as well as the particle-antiparticle pairs created by the potential. Note that at $t = 0$ the density cannot be bounded on a compact support (i.e., it has tails spreading to infinity), a consequence of requiring a single particle wave packet (hence, defined over an expansion containing only contributions from the positive energy sector).

C. Microcausality

Microcausality is the assertion that observables that are spacelike separated commute. While it is frequently considered as an axiom in some versions of QFT [23], microcausality can be explicitly proved for some free quantum fields. In particular the proof that a noninteracting free KG or Dirac field obeys microcausality is a well-known textbook result [24,25]; if $\hat{O}(t,x)$ and $\hat{O}'(t',x')$ are two observables then

$$[\hat{O}'(t',x'), \hat{O}(t,x)] = 0 \tag{14}$$

for $c^2(t'-t)^2 - (x'-x)^2 < 0$. The standard proof involves writing an arbitrary observable as a bilinear combination of field operators,

$$\hat{O}(t,x) = \hat{\Phi}^\dagger(t,x) o(t,x) \hat{\Phi}(t,x), \tag{15}$$

where $o(t,x)$ is a matrix consisting of c-numbers [24,25]. The commutator in Eq. (14) is then written in terms of the anticommutators (for the Dirac case) or commutators (for the KG case) $[\hat{\Phi}^\dagger(t',x'), \hat{\Phi}(t,x)]_\pm$. For free Dirac or KG fields, these anticommutators can be computed explicitly in closed form [24] and are proved to vanish for spacelike separated intervals.

Note that the density operator given by Eq. (10) is the simplest bilinear form involving field operators; this is the only observable we will be interested in in this work.

III. MICROCAUSALITY AND THE IMPOSSIBILITY OF SUPERLUMINAL TUNNELING

Microcausality holds in the presence of a background field. This follows directly from the equal-time commutation relations. A full derivation, when the observable is the density operator, is given in Appendix. In order to connect microcausality with conditions on the wave packet dynamics, consider the following situation (see Fig. 1). We start with a one-particle state $\|\chi\rangle\rangle$ given by Eq. (12). In

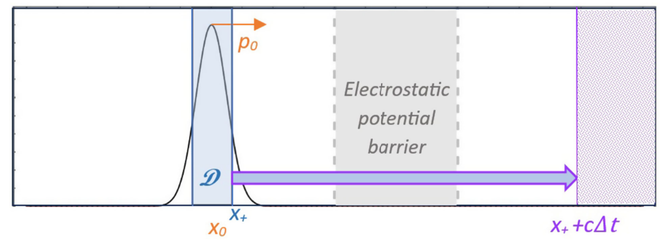

FIG. 1. The initial wave packet at $t_0 = 0$, with average position x_0 and momentum p_0 is shown along with the intervention region $\mathcal{D}$ defined on a compact support. The causally disconnected region at time $t' = \Delta t - t_0$ on the other side of the potential barrier, displayed with purple dots, is defined as the region lying outside the light cone emanating from the right edge x_+ of $\mathcal{D}$.

configuration space, this is a wave packet with mean position x_0 and momentum p_0. As recalled above, the density $\rho(t=0,x) = \langle\langle \chi \| \hat{\rho}(t=0,x) \| \chi \rangle\rangle$ has infinite tails.

From Eq. (12), we can write the first quantized initial wave packet amplitude $\chi(0,x)$ as [26]

$$\hat{\Phi}(0,x) \| \chi \rangle\rangle = \chi(0,x) \| 0 \rangle\rangle, \tag{16}$$

and similarly

$$\langle\langle \chi \| \hat{\Phi}^\dagger(0,x) = \langle\langle 0 \| \chi^\dagger(0,x). \tag{17}$$

Let us introduce a localized intervention on the initial wave packet localized on a compact support $\mathcal{D}$. For definiteness, we will take this intervention to reshape the initial wave packet with the constraint that the total density remains constant. This can be represented by the operator

$$\hat{O}(0,\mathcal{D}) = 1 + \int_{\mathcal{D}} dx f(x) \hat{\Phi}^\dagger(0,x) \hat{\Phi}(0,x), \tag{18}$$

where $f(x)$ is a real function modifying the spatial profile of the initial wave packet. Since the total density on $\mathcal{D}$ is conserved, we have

$$\int_{\mathcal{D}} dx f(x) \chi(x)^\dagger \sigma \chi(x) = 0. \tag{19}$$

Note $\hat{O}$ is bilinear in $\hat{\Phi}$.

We next introduce $\hat{O}'(t',x')$ as being an observable at a space-time point (t',x') lying to the right of the potential barrier and that is spacelike separated relative to any point (t,x) of $\mathcal{D}$. We are interested in the correlation function,

$$\mathcal{C}(t',x';0,x) = \langle\langle \chi \| \hat{O}'(t',x') \hat{O}(0,\mathcal{D}) \| \chi \rangle\rangle, \tag{20}$$

involving the joint operations "intervention on $\mathcal{D}$" and "the application of some observable $\hat{O}'$ at (t',x')". To be specific let us take $\hat{O}'(t',x')$ to be the density $\hat{\rho}(t',x')$. Using Eq. (18) the correlation function becomes

$$\mathcal{C}(t',x';0,x) = \langle\langle\chi\|\hat{\rho}(t',x')\|\chi\rangle\rangle + \langle\langle 0\|\hat{\rho}(t',x')\|0\rangle\rangle \times \int_{\mathcal{D}} dx f(x)\chi^{\dagger}(0,x)\chi(0,x) = \langle\langle\chi\|\hat{\rho}(t',x')\|\chi\rangle\rangle, \quad (21)$$

where we used Eq. (19). Hence, the correlation function is simply the density at (t',x') and does not in any way depend on the intervention carried out at a spacelike interval involving the initial wave packet. If superluminal transmission were possible, one would expect the result of the intervention to be reflected in the transmitted wave packet, in which case Eq. (21) would not hold. We therefore conclude that microcausality prevents superluminal transmission.

As a corollary, consider two initial wave packets $\|\chi\rangle\rangle$ and $\|\tilde{\chi}\rangle\rangle$ that differ inside $\mathcal{D}$ but are identical elsewhere. There is therefore an operator $\hat{O}(0,\mathcal{D})$ defined by Eq. (18) such that $\|\tilde{\chi}\rangle\rangle = \hat{O}(0,\mathcal{D})\|\chi\rangle\rangle$. We then have

$$\langle\langle\tilde{\chi}\|\hat{\rho}(t',x')\|\tilde{\chi}\rangle\rangle = \langle\langle\tilde{\chi}\|\hat{\rho}(t',x')\hat{O}(0,\mathcal{D})\|\chi\rangle\rangle, \quad (22)$$

and using the commutativity of $\hat{O}(0,\mathcal{D})$ and $\hat{\rho}(t',x')$ (as per microcausality) as well as the Hermiticity of $\hat{O}$, the right-hand side term becomes $\langle\langle\chi\|\hat{\rho}(t',x')\|\chi\rangle\rangle$ leading to

$$\langle\langle\tilde{\chi}\|\hat{\rho}(t',x')\|\tilde{\chi}\rangle\rangle = \langle\langle\chi\|\hat{\rho}(t',x')\|\chi\rangle\rangle. \quad (23)$$

This means that for an arbitrary intervention in the region $\mathcal{D}$ at time $t = 0$, the expectation value of the charge density $\rho(t',x')$ at a spacelike distant point from the intervention does not change; the density $\rho(t',x')$ is identical for any wave packets that initially differ only inside $\mathcal{D}$. Note that $\rho(t',x')$ does not vanish due to the particle-antiparticle pairs produced by the barrier, as well as to the dynamics of the tails of the wave packets.

IV. ILLUSTRATIONS

A. Setting

In order to illustrate our main results given in the preceding section within our space-time resolved QFT framework, we undertake here computations of Eq. (23). We specifically start with a Gaussian wave packet of mean position and momentum x_0 and p_0 expressed in Fock space as

$$\|\chi\rangle\rangle = \int dp g_+(p) b_p^{\dagger}\|0\rangle\rangle, \quad (24)$$

with $g_+(p) = \exp(-(p-p_0)^2\sigma^2)\cdot\exp(-ix_0p)$, where σ is the spatial width. The spatial amplitude of this Gaussian wave packet at $t = 0$, introduced in Eq. (16), is denoted here $\chi(x)$. The modified density $\tilde{\chi}(x)$ is obtained by introducing an intervention $F(x)$ on the Gaussian wave packet such that

$$\tilde{\chi}(x) = F(x)\chi(x). \quad (25)$$

$F(x)$ is nonzero only within the compact support $\mathcal{D}$ and is related to the function $f(x)$ of Eq. (18) defining the intervention O by

$$F(x) = \sqrt{1+f(x)}. \quad (26)$$

Note that the QFT state corresponding to the wave packet $\tilde{\chi}(x)$ can be expressed similarly to Eq (24) as

$$\|\tilde{\chi}\rangle\rangle = \int dp \tilde{g}_+(p)\hat{b}^{\dagger}\|0\rangle\rangle. \quad (27)$$

The amplitudes $\tilde{g}_+(p)$ are obtained by Fourier transforming $\tilde{\chi}(x)$. For definiteness we will choose the modified wave packet to result by implementing the intervention

$$f(x) = \sin^{13}(x)\theta\left(x+\frac{D}{2}\right)\theta\left(x-\frac{D}{2}\right) \quad (28)$$

over $\mathcal{D}$ (D is hence the width of $\mathcal{D}$). We will consider a rectangularlike potential barrier given by

$$V(x) = \frac{1}{2}V_0\left(\tanh\left(\frac{x+\frac{d}{2}}{\kappa}\right) - \tanh\left(\frac{x-\frac{d}{2}}{\kappa}\right)\right), \quad (29)$$

where V_0 is the barrier height, d is the barrier width, κ is the smoothness parameter.

In order to compute the densities given by the expectation values $\langle\langle\chi\|\hat{\rho}(t',x')\|\chi\rangle\rangle$ and $\langle\langle\tilde{\chi}\|\hat{\rho}(t',x')\|\tilde{\chi}\rangle\rangle$ we start from Eqs. (10), (11), and (13) in order to obtain the positive and negative charge densities, ρ_+ and ρ_-, and from there the total density ρ. The only difference when computing the densities for each of the two initial wave packets lies in the amplitudes, $g_+(p)$ and $\tilde{g}_+(p)$, respectively. Setting $G(p)$ to indicate either case, we compute the positive charge density from

$$\rho_+(t,x) = \langle\langle 0\| \int dp G^*(p)\hat{b}_p \int dp \hat{b}_p^{\dagger}(t)v_p^{\dagger}(x) \times \sigma \int dp \hat{b}_p(t)v_p(x) \int dp G(p)\hat{b}_p^{\dagger}\|0\rangle\rangle, \quad (30)$$

which after some algebra gives

$$\rho_+(t,x) = \int dp_1 \ldots dp_3 U^*_{v_{p_1}w_{p_2}}(t)v^{\dagger}_{p_1}(x)\sigma U_{v_{p_3}w_{p_2}}(t)v_{p_3}(x) + \int dp_1dp_2 G^*(p_1)U^*_{v_{p_2}v_{p_1}}(t)v^{\dagger}_{p_2}(x) \times\sigma\int dp_1dp_2 U_{v_{p_2}v_{p_1}}(t)v_{p_2}(x)G(p_1); \quad (31)$$

the unitary evolution operator U and the free basis of the Klein-Gordon or Dirac equations have been defined in Sec. II A. Similarly, the density of antifermions or

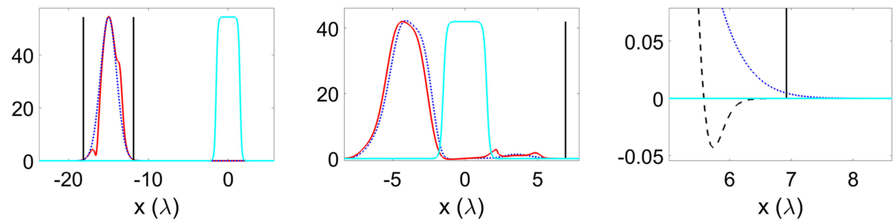

FIG. 2. The left panel shows, in dotted blue, an initial Dirac Gaussian wave packet of width $\sigma = 1\lambda$ ($\lambda = \frac{\hbar}{m_e c}$ is the Compton wavelength of the electron), initial position $x_0 = -15\lambda$, and average momentum $p_0 = \sqrt{V_0^2 - m^2c^4}/c$, where V_0 is the potential barrier height. The mutilated wave packet is shown in solid red, with the intervention function defined by Eq. (28) and $D = 1\lambda$. The vertical black lines mark the edges of the intervention area in the left panel and the light cone emanating from them in the middle and right panels. The potential barrier is given by Eq. (29) with $V_0 = 2.5mc^2$, $\kappa = 0.2\lambda$, and $d = 3\lambda$. The middle panel shows the reflected and tunneled wave packets at $t = 0.137\lambda/c$. In the right panel, we zoom in on the region around the light cone, showing the tunneled part of the Gaussian wave packet in dotted blue and the difference between the Gaussian and the mutilated wave packets in dashed black. The calculations are performed on a lattice of width 100λ, with 2^{11} sites and time steps of $\delta t = 137 \times 10^{-6}\lambda/c$.

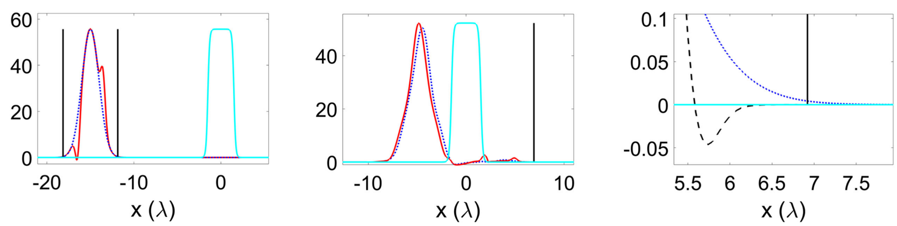

FIG. 3. Initial and tunneled KG wave packets, depicted using the same color scheme as in the Dirac case. The wave packet, barrier, and lattice parameters are identical to those used in the Dirac analysis, with λ now denoting the Compton wavelength of the boson.

antibosons is

$$\rho_-(x) = \epsilon \int dp_1 \ldots dp_3 U^*_{w_{p_1} v_{p_2}}(t) w^\dagger_{p_1}(x) \sigma U_{w_{p_3} v_{p_2}}(t) w_{p_3}(x) - \int dp_1 dp_2 U^*_{w_{p_1} v_{p_2}}(t) w^\dagger_{p_1}(x) G^*(p_2) \times \sigma \int dp_1 dp_3 G(p_1) U_{v_{p_1} w_{p_1}}(t) w_{p_2}(x). \tag{32}$$

In Eqs. (31) and (32), the first term corresponds to the density of created fermions or bosons while the second term corresponds to the time evolution of the wave packet.

B. Numerical results

We now employ the expressions (31) and (32) in order to compute the space-time resolved dynamics first for an initial Gaussian wave packet, and then for that same wave packet but modified inside $\mathcal{D}$ in the setting described above, where the "mutilation" function is given by Eq. (28) and the background potential by Eq. (29). We give an illustration for a Dirac fermion in Fig. 2, and another for a scalar boson described by the Klein-Gordon equation in Fig. 3. The details of the computational method are given elsewhere [15].

In both cases, we can observe that the Gaussian wave packet and the mutilated one coincide outside the light cone, while they differ markedly inside the light cone (see the dashed lines in Figs. 2, 3). This illustrates that the intervention introduced into the initial wave packet has propagated subluminally, with no observable effect outside the light cone.

V. DISCUSSION AND CONCLUSION

We have seen that microcausality implies that the tunneling dynamics must remain causal, precluding superluminal behavior. More specifically, we have proved that an intervention on the initial wave packet density does not modify the density outside the light cone emanating from the region over which the intervention was performed. As a corollary, we obtained that if two initial wave packets have a different density inside a region $\mathcal{D}$ having compact support, the density at a spacelike location from $\mathcal{D}$ is identical for both initial wave packets. We have illustrated these results by carrying out numerical computations of space-time resolved densities for two initial wave packets differing in a region centered on their maximum.

While claims of superluminal or even instantaneous wave packet tunneling times are common in the recent literature, both in experimental and theoretical works, be it within a nonrelativistic or a relativistic framework (see references cited in [13]), such claims conflict with microcausality, a cornerstone of relativistic quantum field theories. Although in some cases identifying a culprit might seem straightforward (in particular many experimental observations rely on nonrelativistic models to set a time-tag), in other cases reconciling causal dynamics with the superluminal behavior of certain quantities (such as the conditional expectation values taken on the transmitted wave packet, or on some instances of group velocities) still needs to be clarified. It would also be interesting to connect our present results to other approaches to QFT tunneling like the use of effective actions or perturbative expansions in an external potential [27,28] or tunneling in the context of false vacuum decay [29,30].

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DATA AVAILABILITY

The data that support the findings of this article are openly available [31], embargo periods may apply.

APPENDIX: MICROCAUSALITY WITH A BACKGROUND FIELD

In the presence of a background field, it is generally impossible to compute the field commutators in the case of bosons (or anticommutators in the case of fermions) $[\hat{\Phi}^\dagger(t',x'),\hat{\Phi}(t,x)]_\pm$ in closed form. However it is straightforward to establish that for spacelike separated points, such commutators (or anticommutators) must vanish. This can be seen by noting that if x and x' are spacelike separated, then there is a reference frame for which x and x' lie in a hypersurface of simultaneity. In this reference frame the commutator for the density becomes $[\hat{\rho}(t,x),\hat{\rho}(t,y)]$ (with $x\neq y$) which can be readily computed as

$$\begin{aligned}[\hat{\rho}(t,x),\hat{\rho}(t,y)] &= \hat{\Phi}^\dagger(t,x)([\hat{\Phi}(t,x),\hat{\Phi}^\dagger(t,y)]_\epsilon\hat{\Phi}(t,y) \\ &\quad + \hat{\Phi}^\dagger(t,y)[\hat{\Phi}(t,x),\hat{\Phi}(t,y)]_\epsilon) \\ &\quad + ([\hat{\Phi}^\dagger(t,x),\hat{\Phi}^\dagger(t,y)]_\epsilon\hat{\Phi}(t,y) \\ &\quad + \hat{\Phi}^\dagger(t,y)[\hat{\Phi}^\dagger(t,x),\hat{\Phi}(t,y)]_\epsilon)\hat{\Phi}(t,x) \\ &= \hat{\Phi}^\dagger(t,x)[\hat{\Phi}(t,x),\hat{\Phi}^\dagger(t,y)]_\epsilon\Phi(t,y) \\ &\quad + \hat{\Phi}^\dagger(t,y)[\hat{\Phi}^\dagger(t,x),\hat{\Phi}(t,y)]_\epsilon\hat{\Phi}(t,x). \end{aligned} \tag{A1}$$

Since $[\hat{\Phi}(t,x),\hat{\Phi}(t,y)]_\epsilon = 0$ due to bosons (fermions) statistics we only need to compute the equal-time commutation (anticommutation) relation,

$$\begin{aligned}[\hat{\Phi}^\dagger(t,x),\hat{\Phi}(t,y)]_\epsilon = \bigg[\int dp(\hat{b}_p^\dagger(t)v_p^\dagger(x)+\hat{d}_p(t)w_p(x)^\dagger) \\ \times\int dp(\hat{b}_p(t)v_p(x)+\hat{d}_p^\dagger(t)w_p(x))\bigg]_\epsilon, \end{aligned} \tag{A2}$$

in the presence of an electromagnetic background field.

This involves the commutators (or anticommutators) of the type

$$\begin{aligned}[\hat{b}_{p_1}^\dagger(t),b_{p_2}(t)]_\epsilon = \bigg[\int dp_1'(U^*_{v_{p_1}v_{p_1'}}\hat{b}^\dagger_{p_1'}+U^*_{v_{p_1}w_{p_1'}}\hat{d}_{p_1'}), \\ \times\int dp_2'(U_{v_{p_2}v_{p_2'}}\hat{b}_{p_2'}+U_{v_{p_2}w_{p_2'}}\hat{d}^\dagger_{p_2'})\bigg]_\epsilon. \end{aligned} \tag{A3}$$

Using Eq. (3), one obtains

$$\begin{aligned}&[\hat{b}_{p_1}^\dagger(t),b_{p_2}(t)]_\epsilon \\ &= \int dp_1'(U^*_{v_{p_1}v_{p_1'}}U_{v_{p_2}v_{p_1'}}\epsilon U^*_{v_{p_1}w_{p_1'}}U_{v_{p_2}w_{p_1'}}) \\ &= \int dp_1'(\langle v_{p_2}|\hat{U}|v_{p_1'}\rangle\langle v_{p_1'}|\hat{U}^\dagger|v_{p_1}\rangle \\ &\quad\times\epsilon\langle v_{p_2}|\hat{U}|w_{p_1'}\rangle\langle w_{p_1'}|\hat{U}^\dagger|v_{p_1}\rangle) \\ &= \langle v_{p_2}|\hat{U}\hat{U}^\dagger|v_{p_1}\rangle = \langle v_{p_2}|v_{p_1}\rangle = \delta(p_1-p_2), \end{aligned} \tag{A4}$$

where in the last line, we used the completeness relation, $\int dp'(|v_{p'}\rangle\langle v_{p'}|+\epsilon|w_{p'}\rangle\langle w_{p'}|) = 1$, with $\epsilon = 1$ in the case of fermions and $\epsilon = -1$ in the case of bosons, and the orthonormality of the solutions of the free Dirac or KG equations. Similarly, one can show that

$$[\hat{d}_{p_1}^\dagger(t),d_{p_2}(t)]_\epsilon = \delta(p_1-p_2). \tag{A5}$$

Inserting these commutators (anticommutators) into Eq. (A2) leads to

$$[\hat{\Phi}^\dagger(t,x),\hat{\Phi}(t,y)]_\epsilon = \int dp(e^{ip(y-x)}+e^{ip(x-y)}) = \delta(x-y). \tag{A6}$$

Pluging-in this result into Eq. (A1) leads to $[\hat{\rho}(t,x),\hat{\rho}(t,y)] = 0$ for $x\neq y$ which ensures $[\hat{\rho}(t,x),\hat{\rho}(t',x')] = 0$ for spacelike separated points by Lorentz transforming back to the original frame. We have therefore shown that microcausality holds for density operators in the presence of a background field.

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