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From observer-dependent facts to frame-dependent measurement records in Wigner friend scenarios

J. Allam and A. Matzkin[(a)]

Laboratoire de Physique Théorique et Modélisation, CNRS Unité 8089, CY Cergy Paris Université 95302 Cergy-Pontoise Cedex, France

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Abstract – The description of Wigner friend scenarios —in which external agents describe a closed laboratory containing a friend making a measurement— remains problematic due to the ambiguous nature of quantum measurements. One option is to endorse assumptions leading to observer-dependent facts, given that the friend's measurement outcome is not defined from the point of view of the external observers. We introduce in this work a model in a relativistic context showing that these assumptions can also lead to measurement records that depend on the inertial reference frame in which the agents make their observations. Our model is based on an entangled pair shared by the friend and a distant agent performing space-like separated measurements. An external observer at rest relative to the closed laboratory and observers in a moving frame do not agree on the observed records, which are not Lorentz transforms of one another.

Introduction. – Accounting for measurements remains one of the most intricate theoretical problems of quantum mechanics. In some instances the theory becomes ambiguous, and unsurprisingly there is no consensus on how to deal with this ambiguity. The main difficulty is how to update the quantum state from a pre-measurement linear superposition to a single term upon performing a measurement.

A well-known instance displaying this ambiguity is the Wigner friend scenario [1] (WFS), that has lately seen a renewed interest (see the review [2] and references therein as well as [3–13]). Originally introduced as a thought experiment, it has recently been argued that a universal quantum computer might give rise to a feasible experiment [14]. The WFS is defined by a (super-)observer W measuring an isolated laboratory in which an agent F (the friend) makes a measurement on a quantum system. The ambivalence is whether the super-observer should update his state upon the friend's measurement, or consider the isolated laboratory as a quantum superposition evolving unitarily.

Many recent works [5,9,11–16] embrace a unitary perspective for W's description. Although these works, undertaken within different interpretative frameworks, take somewhat different views on quantum measurements, they all assume that the quantum state should only be updated when an agent observes a measurement record. So in a Wigner friend scenario F measures the system and updates her state (describing the measured system and the apparatus), but for W the entire isolated laboratory remains a closed quantum system and hence evolves unitarily. Facts then become observer dependent, as F and W's account and records of the quantum system measured by F are different.

Another instance in which updating a quantum state after a measurement is ambiguous happens when relativistic considerations are taken into account. For example if Alice (A) and Bob (B) measure an entangled pair at space-like separated locations, in one reference frame A measures first and the state is updated before B measures, while there are other inertial frames in which the time ordering of the measurements is inverted and the state is updated and disentangled before A measures her particle. The current consensus [17–19] is that state update can be assumed to take place instantaneously in any reference frame. Having different quantum state assignments in different frames is inconsequential —wave functions do not transform covariantly when measurements occur [20]; what only matters is that the measurement outcomes and their probabilities remain Lorentz transforms of one another, irrespective of the choice of hypersurface on which the state is updated.

In this work we will show that introducing relativistic considerations in unitary accounts of WFS leads to frame-dependent outcomes. This will be done on the basis of

(a) E-mail: alexandre.matzkin@u-cergy.fr (corresponding author)

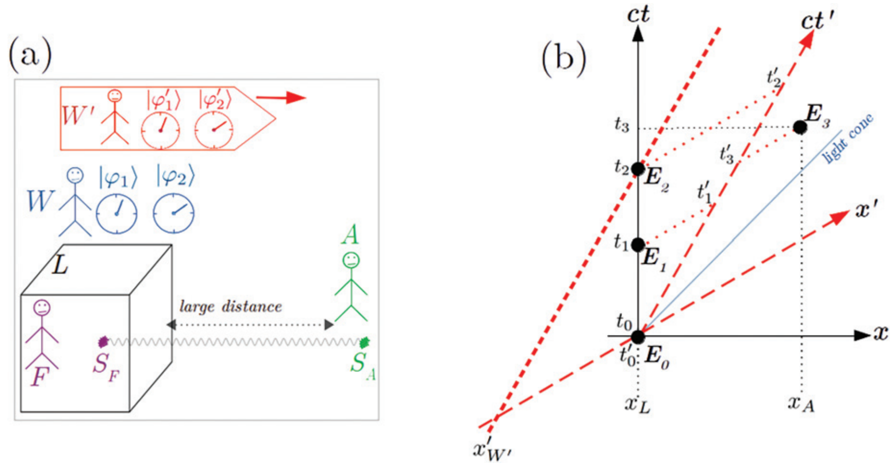

Fig. 1: (a) The scenario involves a friend F inside a sealed laboratory L, sharing an entangled spin-$\frac{1}{2}$ pair with a distant observer A. F measures her particle S_F and then sends 2 qubits outside the lab through a special channel. These qubits are then measured by an observer W (in $\mathcal{R}$) or W′ (in $\mathcal{R}'$), either through a weak measurement, or, in another version of the protocol, by a standard projective measurement. (b) Spacetime diagram (not to scale) showing events $E_0 - E_3$ with respect to reference frames $\mathcal{R}$ (solid axes $(ct,\, x)$) and $\mathcal{R}'$ (dashed axes $(ct',\, x')$). The time of event E_i in each reference frame is shown along with the space-like hypersurface in that reference frame (dotted lines). The positions of x_L and x_A of the lab L and agent A, respectively, are also shown, as well as the position of W′, fixed in $\mathcal{R}'$.

a specific model. We will first introduce a probabilistic scenario in which W and W′, external observers in two reference frames $\mathcal{R}$ and $\mathcal{R}'$, perform weak measurements [21,22] on qubits correlated with the state of the laboratory. The average position of the pointers is different in $\mathcal{R}$ and $\mathcal{R}'$, leading W and W′ to disagree on their observations and also on their assignments concerning the state of the laboratory. We will then describe more briefly a slightly different scheme not relying on statistical averages. In both cases we will require measurement outcomes to be grounded on the existence of physical records [23]. We will finally discuss the results. Let us anticipate here that we do not expect frame-dependent facts to be physically possible, so that our results would rather point to a problem arising when the two assumptions of unitary evolution for the friend and instantaneous state update in any reference frame are put together. Note we will neglect throughout the relativistic character of the spin and the resulting frame dependence of entanglement [24], thereby assuming a low velocity for the moving frame.

A Wigner friend protocol. – In our scenario, we consider a friend F in a sealed laboratory L, a super-observer W sitting next to L, and a distant observer A (for Alice), at rest in $\mathcal{R}$ (see fig. 1(a)). In this reference frame, L is positioned at $x = 0$, W sits next to L, and A is at $x = x_A$, see fig. 1(b). A and F share an entangled state, say $|\psi\rangle = \alpha\,|+\rangle_F\,|+\rangle_A + \beta\,|-\rangle_F\,|-\rangle_A$, where $\sigma_z\,|\pm\rangle = \pm\,|\pm\rangle$ (we set $\hbar = 1$). We will set $\alpha = \beta = 1/\sqrt{2}$ for simplicity. Inside the friend's laboratory, we will discriminate the particle whose spin is to be measured denoted as $|\pm\rangle_F$, the measurement apparatus with pointer states $|m_k\rangle$, and the other degrees of freedom collectively represented by the environment states $|\varepsilon_k\rangle$. W will make weak measurements on two qubits and require for that two pointers initially in state $|\varphi_1\rangle$ and $|\varphi_2\rangle$ (the wave functions $\varphi_i(X_i)$ can be taken to be Gaussians). E_0 is the initial event corresponding to preparation, at $t = 0$ in $\mathcal{R}$ and with the quantum state given by

$$|\Psi(t=0)\rangle = |\psi\rangle\,|m_0\rangle\,|\varepsilon_0\rangle\,|\varphi_1\rangle\,|\varphi_2\rangle. \tag{1}$$

Event E_1 corresponds to the friend's measurement: at $t = t_1$ the friend measures the spin component along z by coupling her pointer to the particle (see fig. 2 for the time ordering of the different steps of the protocol). The quantum state becomes

$$|\Psi(t_1)\rangle = \alpha\,|L_+\rangle_F\,|+\rangle_A + \beta\,|L_-\rangle_F\,|-\rangle_A, \tag{2}$$

where $|L_\pm\rangle \equiv |\pm\rangle_F\,|m_\pm\rangle\,|\varepsilon_\pm\rangle$ denotes the quantum state of the entire isolated laboratory (we do not explicitly write the pointer states as they remain unchanged). Then the friend immediately resets the states of her spin and the measuring device to a pre-assigned arbitrary state $|s_0\rangle|m_0\rangle$. Defining $|\tilde{L}_\pm\rangle \equiv |s_0\rangle|m_0\rangle|\varepsilon_\pm\rangle$, the quantum state thus becomes

$$|\Psi(t_1)\rangle = \alpha|\tilde{L}_+\rangle_F\,|+\rangle_A + \beta|\tilde{L}_-\rangle_F\,|-\rangle_A. \tag{3}$$

Recall that there is no state update despite F's measurement, since eqs. (2) and (3) represent the evolution of the entire laboratory for external observers, including W and A. The friend can open a channel and communicate to W the fact that she obtained a measurement outcome, *e.g.*, with an ancilla qubit [12,25] initially in state $|0\rangle$ and becoming $|1\rangle$ after she completed her measurement, as long as the state of the ancilla does not get entangled

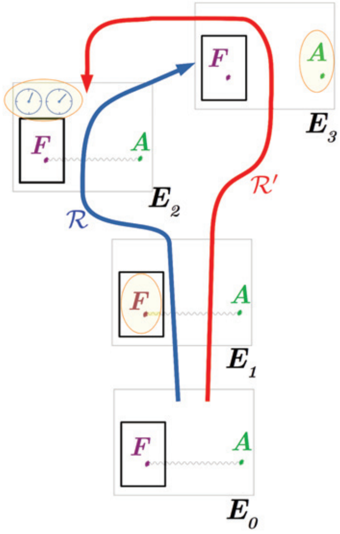

Fig. 2: Time ordering of events E_0, E_1, E_2, E_3 in the two reference frames $\mathcal{R}$ (represented by the blue arrow) and $\mathcal{R}'$ (represented by the red arrow). Event E_0 represents preparation (see fig. 1). E_1 corresponds to F in her sealed lab measuring her qubit and resetting its state. Event E_2 represents F sending 2 qubits outside her lab, where they are immediately measured by an observer (W or W′) and E_3 corresponds to A measuring her qubit. Note that the time ordering of E_2 and E_3 is inverted in $\mathcal{R}$ relative to $\mathcal{R}'$. After several runs in which either W or W′ performs a weak measurement of the qubits, the average displacement of the pointers weakly coupled to the qubits can be determined.

with L (in particular, the outcome obtained cannot be communicated).

We label by E_2 the following event: at time t_2, F creates 2 particles sent to W by opening a communication channel. The particles sent to W are qubits prepared in a state identical to the one observed by F after her previous measurement at $t = t_1$. Note that for her observation F, a macroscopic agent, relies on the environment states $|\varepsilon_\pm\rangle$. The state after the qubits are created[1] and sent to W becomes

$$|\Psi(t_2)\rangle = (\alpha|\tilde{L}_+\rangle_F |+\rangle |+\rangle |+\rangle_A + \beta|\tilde{L}_-\rangle_F |-\rangle |-\rangle |-\rangle_A) |\varphi_1\rangle |\varphi_2\rangle. \quad (4)$$

W immediately makes a weak measurement of the two qubits: first a spin observable O is coupled to the momentum P_i of each pointer i, *i.e.*, we apply the unitary $\exp(-igOP_1)\exp(-igOP_2)$ to $|\Psi(t_2)\rangle$ where g is the coupling constant between each qubit and its pointer. Then each qubit is post-selected by making a projective measurement of σ_{θ_1} and σ_{θ_2} on qubits 1 and 2, respectively, and filtering the positive outcome in each case (we define $|+\theta_i\rangle = \cos\frac{\theta_i}{2}|+\rangle + \sin\frac{\theta_i}{2}|-\rangle$ and $\sigma_{\theta_i} = \overrightarrow{\sigma}\cdot\overrightarrow{n}_i$, where $\overrightarrow{n}_i$ makes an angle θ_i with the z-axis and lies in the xz-plane). When g is small (weak coupling) the unitaries can be expanded to first order leading to an expression that is usually formalized [21,22] as

$$\begin{aligned}&\langle+\theta_1|\langle+\theta_2|\,\Psi(t_2)\rangle = \\ &\alpha|\tilde{L}_+\rangle_F|+\rangle_A\langle+\theta_1|+\rangle\langle+\theta_2|+\rangle\left|\varphi_1^+\right\rangle\left|\varphi_2^+\right\rangle \\ &\quad+\beta|\tilde{L}_-\rangle_F|-\rangle_A\langle+\theta_1|-\rangle\langle+\theta_2|-\rangle\left|\varphi_1^-\right\rangle\left|\varphi_2^-\right\rangle,\end{aligned} \quad (5)$$

where the pointer states $\left|\varphi_i^\pm\right\rangle$ are the initial pointer states shifted by the weak values $\sigma_i^\pm$, *e.g.*,

$$\left|\varphi_1^-\right\rangle = e^{-igP_1\sigma_1^-}|\varphi_1\rangle \text{ with } \sigma_1^- \equiv \frac{\langle+\theta_1|O|-\rangle}{\langle+\theta_1|-\rangle}, \quad (6)$$

and similarly in the other cases. For definiteness we will set $O = \sigma_z$, so that the z spin component is measured weakly for both qubits.

Right after post-selection (we neglect the duration τ of the weak measurement [22]) W measures the position X_1 and X_2 of each pointer. By repeating the experiment, he collects statistics (when post-selection is successful) to determine the average pointer positions. Theoretically the average is obtained from eq. (4) by tracing out the friend's and Alice's degrees of freedom in order to obtain the reduced density matrix ρ_{12}. Recalling that by construction the average position is $\int X_i\left|\varphi_i^\pm(X_i)\right|^2 \mathrm{d}X_i = g\sigma_i^\pm$, and labeling the post-selection projectors $\Pi_{+\theta_i} \equiv |+\theta_i\rangle\langle+\theta_i|$ we obtain for the choices made here

$$\langle X_1X_2\rangle = \mathrm{Tr}\left(\rho_{12}\Pi_{+\theta_1}\Pi_{+\theta_2}X_1X_2\right) = \frac{g^2}{4}\left(1+\cos\theta_1\cos\theta_2\right). \quad (7)$$

Note that this result is obtained by W by observing his own pointers, irrespective of the outcomes that can be attributed to F. Finally the event E_3, at t_3 in $\mathcal{R}$ corresponds to A measuring her qubit in the $|\pm x\rangle$ basis ($|\pm x\rangle = (|+\rangle \pm |-\rangle)/\sqrt{2}$); she obtains her local predictions for her qubit's outcome by tracing out all the other degrees of freedom.

Let us now consider the same protocol but for observers in a different inertial frame $\mathcal{R}'$ (see fig. 1). $\mathcal{R}'$ is chosen such that E_3 happens before E_2, *i.e.*, A measures her spin before F sends her qubits outside the lab. The time ordering is $t_0' < t_1' < t_3' < t_2'$ (see fig. 2). F makes her measurement first followed by spin reset, as per eqs. (2), (3). But after A's measurement $|\Psi'(t_1')\rangle = \alpha|\tilde{L}_+\rangle_F|+\rangle_A + \beta|\tilde{L}_-\rangle_F|-\rangle_A$ is updated to either

$$|\Psi'_{+x}(t_3')\rangle = \frac{1}{\sqrt{2}}|s_0\rangle|m_0\rangle\left(|\varepsilon_+\rangle + |\varepsilon_-\rangle\right) \equiv |s_0\rangle|m_0\rangle|\varepsilon_{+x}\rangle \quad (8)$$

or to

$$|\Psi'_{-x}(t_3')\rangle = \frac{1}{\sqrt{2}}|s_0\rangle|m_0\rangle\left(|\varepsilon_+\rangle - |\varepsilon_-\rangle\right) \equiv |s_0\rangle|m_0\rangle|\varepsilon_{-x}\rangle. \quad (9)$$

We have used the notation $|\varepsilon_{\pm x}\rangle \equiv \left(|\varepsilon_+\rangle \pm |\varepsilon_-\rangle\right)/\sqrt{2}$ to emphasize that the superposition of the environment

[1]Formally we can introduce a vacuum state $|0\rangle$ in eq. (2) and consider particle creation operators at the point r, $\Phi_\pm^\dagger(r) = \int \mathrm{d}k a^\dagger(k)e^{ikr}c(k)$ acting on $|0\rangle$. $\Phi_\pm^\dagger(r)$ creates a particle wavepacket with the spatial profile determined by $c(k)$ and spin $\pm$.

states $|\varepsilon_+\rangle \pm |\varepsilon_-\rangle$ can be taken to represent the state of the environment that would be obtained had σ_x been measured[2]. Note however that in order to run our argument we just need $|\varepsilon_{\pm x}\rangle$ to be macroscopically different from $|\varepsilon\pm\rangle$. Indeed the friend is also a macroscopic object and any observation of a quantum result is distilled by the environment [26]. Consequently, the superpositions (8), (9) resulting in either $|\varepsilon_{+x}\rangle$ or $|\varepsilon_{-x}\rangle$ correspond to a state in which the friend now observes a spin outcome different from the original measurement basis. The records of the original measurement of σ_z, encapsulated in the environment states $|\varepsilon_\pm\rangle$ have been erased [16,23] by the superpositions, eqs. (8), (9), which can be rewritten as

$$|\Psi'_{\pm x}(t'_3)\rangle = |\tilde{L}_{\pm x}\rangle. \tag{10}$$

Following our protocol, E_2 takes place at time t'_2: F opens a channel and sends the 2 qubits outside L to W′, an observer at rest in $\mathcal{R}'$ and moving with respect to F. In $\mathcal{R}'$ the state updates (8) and (9) imply that the qubits sent by F will be in accordance with her observation at t'_3, namely $|+x\rangle$ if the environment is in state $|\varepsilon_{+x}\rangle$ or $|-x\rangle$ if the environment is in state $|\varepsilon_{-x}\rangle$. The resulting quantum state at the time W′ measures the qubits is

$$|\Psi'_{+x}(t'_2)\rangle = |\tilde{L}_{+x}\rangle\,|+x\rangle\,|+x\rangle, \tag{11}$$

or

$$|\Psi'_{-x}(t'_2)\rangle = |\tilde{L}_{-x}\rangle|-x\rangle\,|-x\rangle. \tag{12}$$

After the weak measurements of the 2 qubits by W′, an observer in $\mathcal{R}'$ will describe the shifted pointer states as either $\left|\varphi_1'^{+x}\right\rangle\left|\varphi_2'^{+x}\right\rangle$ or $\left|\varphi_1'^{-x}\right\rangle\left|\varphi_2'^{-x}\right\rangle$ where, for example,

$$\left|\varphi_1'^{-x}\right\rangle = e^{-igP_1'\sigma_1^{-x}}\,|\varphi_1'\rangle \quad \text{with } \sigma_1^{-x} \equiv \frac{\langle +\theta_1|\,O\,|-x\rangle}{\langle +\theta_1|\,-x\rangle}, \tag{13}$$

and similarly for the other pointer states. Note that the pointer shifts in $\mathcal{R}$ and $\mathcal{R}'$ are different (compare eqs. (6) and (13)). The average shifts will also be different; from the mixed density matrix, and recalling that we have taken $O = \sigma_z$ (and that we have assumed the spin is not affected by Lorentz transformations), we obtain

$$\langle X_1' X_2'\rangle = g^2 \cos\theta_1 \cos\theta_2. \tag{14}$$

This is different from the average observed in $\mathcal{R}$. The difference does not depend on any parameters characterizing the Lorentz transformation between the two frames but on the fact that F sends qubits that are different in each frame. Having frame-dependent averages can hardly lead to a consistent picture, as will perhaps be clear by slightly modifying our scenario in order to obtain the frame-dependent character without relying on statistics.

[2]This can be seen, for example, if the friend measures σ_z on a spin in state $|+x\rangle$, resets the pointer to $|m_0\rangle$, and measures σ_x. For an external observer the lab is left in a superposition of states $|\pm x\rangle\,|m_0\rangle\,(\langle\pm x|\,+\rangle\,|\varepsilon_+\rangle + \langle\pm x|\,-\rangle\,|\varepsilon_-\rangle)$. If F deletes all records of the previous σ_z measurement, the term between $(\ldots)$ must correspond to $|\varepsilon_{\pm x}\rangle$.

A modified scenario with projective measurements. – In the modified protocol, W (or W′) does not perform a weak measurement on each qubit sent by F, but makes a standard projective measurement on each qubit, one in the $|\pm\rangle$ basis, the other in the $|\pm x\rangle$ basis. In $\mathcal{R}$, we see from eq. (4) that this breaks the superposition of lab states $|\tilde{L}_\pm\rangle$ and leads to a definite state for F's environment, either $|\varepsilon_+\rangle$ or $|\varepsilon_-\rangle$ in which the physical record is embedded. F can declare the measurement outcome obtained at t_1, + or −, for example by sending a huge number of qubits in states $|+\rangle$ or $|-\rangle$ that W can analyze. Hence, F's outcome that was defined solely for F in the time interval $t_1 < t < t_2$ becomes public and objective for observers outside the lab at $t > t_2$. Again, Alice measures her qubit at t_3 but this does not change F's record, since the lab was opened and the state update took place at t_2.

In $\mathcal{R}'$, A's measurement takes place before F sends her qubits, so that W′'s projective measurements on the qubits take place after the state update. From eqs. (11), (12), we see that W′'s qubits measurements leave again the lab environment in a definite state but this time for an observer in $\mathcal{R}'$ the assigned state is $|\tilde{L}_{+x}\rangle$ or $|\tilde{L}_{-x}\rangle$. At this stage F can declare her outcome in a similar way.

This modified protocol therefore leads to different physical records being observed in $\mathcal{R}$ and $\mathcal{R}'$, which as mentioned above is a consequence of the unitary quantum description of an agent making interventions that precisely depend on the quantum state at intermediate times, a state that is different in the two reference frames. Note that we now rely on the friend's outcomes, assuming the laboratory can be opened unambiguously if it is in a definite eigenstate, at which stage F's record becomes objective. In the statistical version of the protocol the frame-dependent outcomes were the external weakly coupled pointers. In neither case do we assume that the laboratory is directly measured by W or W′.

Discussion and conclusion. – While dealing with observer-dependent facts might be considered to be a viable option (it is consistent since the observers are isolated from one another and the alternative facts cannot be defined jointly [15]), having facts depending on an observer's reference frame cannot be accepted; for example, for the present protocol it can lead to signalling [27]. Evidently, relaxing the assumption that the laboratory can be described by a quantum state or, if this is deemed possible, relaxing the hypothesis that (for practical or fundamental reasons) its evolution can be described unitarily avoids the inconsistency. Note that in the latter case, this means that a measurement by an observer leads all the other agents to update their state assignments (to a mixed density matrix). In this case, it can be readily verified that although the quantum states at intermediate times will be different in $\mathcal{R}$ and $\mathcal{R}'$, the observers in all reference frames will agree on the outcomes and probabilities of each event (in general the density matrices are then connected by a Lorentz transformation).

Notwithstanding, it is worthwhile to examine the possibility that more specific assumptions could play a role. First, while an isolated macroscopic system might well be accounted for unitarily, demanding an agent's arbitrary operations to be described with unitaries implies stronger constraints. Indeed it has been argued that a valid measurement (as opposed to the creation of correlations) cannot rely on a global unitary evolution —it requires an "intervention" [28]. Hence certain operations realized by an agent might not be possible to model by assuming a perfect correlation between the state of the measured spin and the quantum state of the laboratory. Second, when dealing with entangled states, we have treated the laboratory, a very massive complex system, the same way we would account for entangled elementary particles, such as photons or electrons. This might not be correct, either for practical reasons due to decoherence [29], or for fundamental reasons, *e.g.*, if, as advocated by Bell [30,31], there is a preferred quantum frame [32] that would change the state update rule or the entanglement properties in a mass-dependent way.

To sum up, we have introduced a model for a Wigner friend scenario for which assuming the friend and Wigner observe different facts also leads to observers in different reference frames disagreeing on their observations. In a statistical version of the model, an external observer in one or the other reference frame performs weak measurements on qubits correlated with the state of the laboratory and the average state of the qubit pointers was seen to depend on the reference frame. In a one-shot version of the model the external observer indirectly opens the laboratory by making projective measurements on the qubits, and this leads the friend to declare outcomes that are different in each reference frame. We briefly discussed which assumptions could be relaxed in order to avoid frame-dependent results. More work is needed in order to understand how to account quantum mechanically for physical operations carried out by agents, especially in view of future experimental realizations of such scenarios with quantum computers.

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