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Effects of a transient barrier on wavepacket traversal

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Abstract

An analytical treatment of a propagating wavepacket incident on a transient barrier reveals an effect in which, for a particular time interval, the time-varying transmission probability exceeds that for the free propagation of the wavepacket. We show that this effect can be interpreted semiclassically. This effect is quantified and it is shown that its magnitude is in one-to-one correspondence with the strength of the barrier, a feature that has the potential to be used in a scheme for key generation. It is found that the speed with which the information about the barrier perturbation propagates across the wavepacket can exceed the group velocity of the wavepacket. An application to the speed-up of entanglement generation is also considered.

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(Some figures may appear in colour only in the online journal)

1. Introduction

A number of interesting phenomena have been uncovered in recent years using the dynamics of quantum wavepackets. Among these, a class of novel effects as in the revival of wavepackets [1] and quantum transients [2] are worth mentioning. In particular, for propagating wavepackets, appropriate changes in the boundary conditions for suitable potentials can give rise to curious dynamical features [3–6]. One such effect, a version of which was numerically put into evidence in earlier works [5, 6] and called 'quantum superarrival', is demonstrated in this paper in terms of an analytical treatment by considering a Gaussian wavepacket which is incident on a time-dependent potential barrier. For the purpose of the treatment given here, the form of this barrier is chosen such that it corresponds to a transient parabolic barrier acting over a small time interval during which the peak of the propagating wavepacket crosses the maximum of the parabolic barrier. In this case, we confirm the existence of an interval of time during which there is an enhancement of the time-evolving transmission probability as compared to the case of a wavepacket freely propagating in the absence of any potential barrier. We further show that this effect can be explained semiclassically by analyzing the underlying classical dynamics encapsulated in the path integral form of the evolution operator.

The term 'superarrival' used in [5, 6] refers precisely to this early increase (relative to the free case) in the transmission probability. In the usual studies, the transmission/reflected probabilities for the scattering of wavepackets by potential barriers are calculated after a complete time evolution when the asymptotic values have been attained. In this work, based on the analytical solution of the relevant time-dependent Schrödinger equation, a phenomenon is displayed which occurs during the time evolution of such a probability that is found to have the following salient features. While the effect of barrier perturbation resulting in early arrivals is discernible by measuring the transmission probability, it becomes more pronounced with the increase of the rate at which the barrier perturbation occurs (in the case considered, it is the strength of the barrier). Furthermore, it is shown that the effect of barrier perturbation propagates across the wavepacket at a speed that depends upon the strength of the barrier, thereby leading to the notion of *'information velocity'*.

In particular, for appropriate choices of the relevant parameters, it is shown that this information velocity can be higher than the group velocity of the incident wavepacket, thereby leading to earlier arrival times. Here, a local change in the potential affects a wavepacket globally through its time evolution where the wavefunction plays the role of a *carrier* through which the information about the barrier perturbation is transmitted. Interestingly, by exploiting this feature, there is the possibility to develop a scheme for communication whose basic idea is indicated in this paper. For this, we proceed by first delineating the relevant details of the analytical treatment that leads to earlier arrival times (section 2). We then give a dynamical interpretation of the phenomenon in terms of the path integral propagator (section 3), before considering applications to communication and entanglement generation, as discussed in section 4.

2. Transmission through a time-dependent parabolic barrier

2.1. Time-dependent solutions

We begin our analysis by considering a Gaussian wavepacket peaked at q_0:

$$\psi(x, t_0) = \left(\frac{2m}{\pi\alpha_0^2}\right)^{1/4} \mathrm{e}^{-m[x-q_0]^2/\alpha_0^2} \mathrm{e}^{\mathrm{i}p_0[x-q_0]/\hbar} \tag{1}$$

which is incident on a time-dependent barrier given by

$$V(x, t) = -\frac{1}{2} mk\, \mathrm{e}^{-g(t-t_B)^2} x^2, \tag{2}$$

that corresponds to the appearance of a parabolic barrier during a small time interval. This is achieved by choosing a Gaussian form for the time window, with the parameters t_B and g indicating the peak time and inverse width of the window, respectively. k determines the barrier strength.

The solutions of the Schrödinger equation for the time-dependent parabolic barrier can be obtained exactly by employing different methods, such as algebraic methods [7, 8], dynamical invariant methods [9] or path-integral propagator methods [10]. These methods are usually transposed from the better known time-dependent harmonic oscillator problem (see [11] and references therein). Here we look for the solutions by starting from the ansatz

$$\psi(x,t) = \left(\frac{2m}{\pi\alpha^2(t)}\right)^{1/4} \mathrm{e}^{-[x-q(t)]^2\left(\frac{m}{\alpha(t)^2}-\frac{\mathrm{i}m\alpha'(t)}{2\hbar\alpha(t)}\right)} \mathrm{e}^{\mathrm{i}p(t)[x-q(t)]/\hbar}\, \mathrm{e}^{\frac{\mathrm{i}}{2\hbar}[p(t)q(t)-p_0q_0]}\, \mathrm{e}^{\frac{-\mathrm{i}}{2}[\phi(t)-\phi_0]}, \tag{3}$$

obtained by combining the path-integral properties for quadratic Lagrangians (discussed below) with the properties of Ermakov systems. Ermakov systems involve dynamical invariants (in the form of the 'Ermakov invariant') while allowing to make a straightforward connection with classical equations of motion (see [12] and references therein). It can be checked by direct substitution that equation (3) obeys the Schrödinger equation with the initial condition (1), provided $q(t)$ and $p(t)$ obey the classical equations of motion, i.e.

$$\partial_t^2 q(t) = \omega^2(t)q(t) \tag{4}$$

and $\partial_t p(t) = m\partial_t^2 q(t)$, with $q_0 \equiv q(t_0)$ and $p_0 \equiv p(t_0)$; $\alpha(t)$ is a solution of the nonlinear equation

$$\frac{\partial_t^2\alpha(t)}{\alpha(t)} - \omega^2(t) = \frac{4\hbar^2}{\alpha^4(t)}. \tag{5}$$

This nonlinear equation forms with the linear equation (4) an Ermakov pair [12]. This means that $\alpha(t)$ can be expressed in terms of two linearly independent solutions of equation (4), the precise choice of a given function $\alpha(t)$ depending on two arbitrary constants (denoted I and c in [12]). These are fixed so that initially $\alpha(t_0) = \alpha_0$ and $\alpha'(t_0) = 0$, as required so that equation (1) is consistent with equation (3). One then has $q(t) = \sqrt{2I}\alpha(t)\sin\phi(t)$, where $\phi(t)$ which appears in equation (3) is known in the context of Ermakov systems as the phase function; it is given by $\partial_t\phi(t) \equiv 2\hbar\alpha^{-2}(t)$. Note that Ermakov systems have often been employed in order to study the solutions of the classical time-dependent harmonic oscillator [11]. Besides transforming an ubiquitous nonlinear equation into a linear one, they offer several advantages: for example by construction $\alpha(t)$ is a positive definite quadratic form [12], ensuring that $\psi(x,t)$ given by equation (3) is normalizable.

2.2. A measure of early arrivals

We consider situations in which an initial Gaussian wavefunction (1) lies far on the negative axis and is launched at t_0 toward the right. The wavepacket spreads while traveling to the right (with the spread controlled by $\alpha(t)$). A detector placed at a point x_T far beyond q_0 measures the time-dependent transmission probability by counting the transmitted particles arriving there up to various instants. At any instant *before* the asymptotic value of the reflection probability is attained, the time evolving transmission probability in the region $x_T \leqslant x < \infty$ is given by

$$T(x_T,t) = \int_{x_T}^{\infty} |\psi(x,t)|^2 \mathrm{d}x \tag{6}$$

and then it follows from equation (3) that

$$T(x_T,t) = \frac{1}{2}\mathrm{erfc}\left[\frac{\sqrt{2m}(x_T - q(t))}{\alpha(t)}\right]. \tag{7}$$

For definiteness we will take $T(x_T,t)$ to describe the probability of the arrival of the wavepacket at $x = x_T$ at time t. Recall that the notion of arrival times in quantum mechanics is ambiguous [13] and that strictly speaking $T(x_T,t)$ corresponds to the probability that the particle is found at $x > x_T$ when performing a position measurement.

We compute the transmission probabilities for various sets of parameters. In order to assess the influence of the appearance of the time-dependent barrier on the transmitted wavepacket, we set t_B in equation (2) so that the maximum of the free Gaussian reaches $x = 0$ when

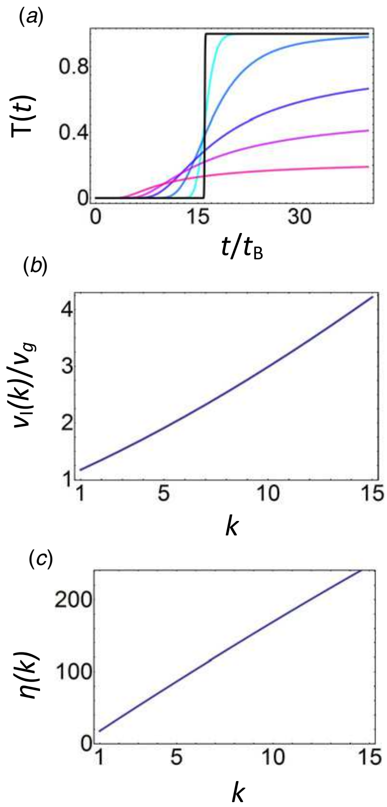

Figure 1. Features of wavepacket transmission for a system with parameters $q_0 = -10^3$, $p_0 = 10$, $m = 5 \cdot 10^4$, $\alpha^2(t_0) = 10^7$, $t_B = 5 \cdot 10^6$, $g = 10^{-10}$ (atomic units). (*a*) The transmission probability $T(t)$ is plotted for the free case (black curve) and for several time-dependent barriers characterized by different strengths: $k/10^{11} = 1, 3, 6, 9, 15$ (the curve color goes from light blue to red (from top to bottom) as k increases). (*b*) The ratio $v_I(k)/v_g$ between the barrier perturbed information velocity and the free group velocity is plotted as a function of the barrier strength k. (*c*) The magnitude of early arrivals η is plotted versus k. For the values of k considered in (*a*), t_d/t_B ranges from 4.55 to 13.80, whereas t_c/t_B ranges from 15.94 to 15.99.

the barrier strength is the greatest, i.e. $V(x, t_B) = -\frac{1}{2}mkx^2$ and $q_f(t_B) = 0$ where f denotes the free case ($k = 0$). Taking identical initial Gaussians in the free and barrier cases, we can appropriately choose the initial position and momentum parameters of the initial wavefunction so that $\psi(x, t)$ and $\psi_f(x, t)$ remain almost identical up to times slightly below $t = t_B$. At that point, the rising barrier perturbs the wavepacket, whereas in the free case the Gaussian keeps propagating with an average momentum $p_f(t) = p_0$. Note that depending on the barrier height k and width g, part of the wavepacket can be reflected; the transmitted and reflected parts are described by the right and left tails of equation (3) respectively, that spread as t increases.

The transmission probabilities are plotted (as a function of time) in figure 1(*a*) in the free case (black curve) and for barriers with increasing strength (as the coloring goes from blue to red). We denote the transmitted probability for the free and the barrier-perturbed cases as $T_f(t)$ and $T_k(t)$, respectively. We observe that $T_k(t) > T_f(t)$ during the time interval $t_d < t < t_c$ (corresponding to early arrival times when compared to free propagation). Here t_B is taken as the instant at which the perturbation starts, t_c is the instant when the free and the perturbed curves cross each other and t_d is the time from which the curve corresponding to the perturbed

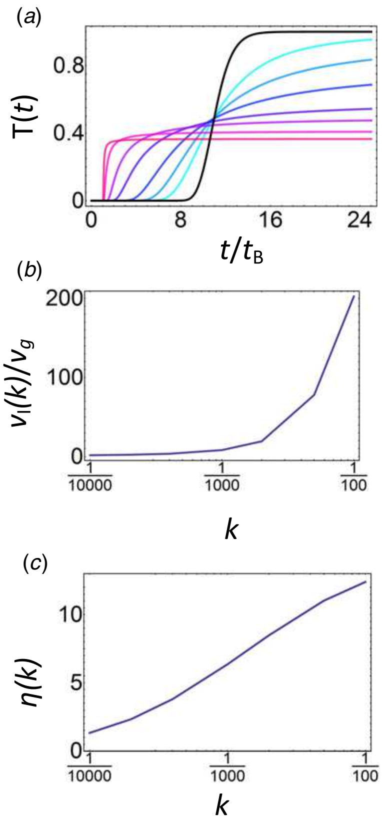

Figure 2. Same as figure 1 but for a system with parameters $q_0 = -10^3$, $p_0 = 2$, $m = 1$, $\alpha^2(t_0) = 5$, $t_B = 500$, $g = 1/500$ (atomic units). The transmission probability $T(t)$ is plotted for the free case (black curve) and for several time-dependent barriers characterized by the strengths: $k = 1/10\,000, 1/5000, 1/2500, 1/1000, 1/500, 1/200, 1/100$ (the curve color goes from light blue curve to red (from top to bottom) as k increases). For the values of k considered in (*a*), t_d/t_B ranges from 1.05 to 6.44, whereas t_c/t_B ranges from 10.59 to 10.98.

case starts deviating from that in the free case, so that $t_c > t_d > t_B$. t_d is determined as the time for which the signal strength reaches a given threshold; experimentally this is related to the detector efficiency (for the numerical applications given in this work we have set this threshold to be 1% of the maximum signal strength).

We now define the quantity η which determines the magnitude of this effect given by

$$\eta(k) = \frac{I_k - I_f}{I_f}, \tag{8}$$

where I_k and I_f are defined with respect to the time interval $\Delta t = t_c - t_d$ during which superarrival occurs, as follows:

$$I_k = \int_{\Delta t} T_k(t)\,\mathrm{d}t; \quad I_f = \int_{\Delta t} T_f(t)\,\mathrm{d}t. \tag{9}$$

The magnitude of early arrival η is a function of the barrier strength k. In figures 1(*c*) and 2(*c*) we plot η versus k for a couple of different sets of parameter values.

Next, in order to determine how fast the information about barrier perturbation travels to the detector, we note that the observer who records the growth of the transmission probability becomes aware of the perturbation (occurring from the instant t_B) at the instant t_d when the transmission probability starts deviating from the free case. If the potential barrier and the detector are separated by the distance D, one can define[4] an information velocity v_I by

$$v_I(k) = \frac{D}{t_d - t_B}. \tag{10}$$

We compute $v_I(k)$ and plot the function $v_I(k)/v_g$ (where v_g refers to the group velocity of the wavepacket in the free case) versus the strength of the barrier (figure 1(*b*)). Note that information of barrier perturbation travels from the barrier to the detector with a velocity which could exceed the group velocity of the wavepacket. Figure 2 displays features of early arrivals with parameters chosen so as to enhance the ratio $v_I(k)/v_g$ (see figure 2(*b*)).

3. Dynamical interpretation

The arrival times are modified by the diffraction of the wavefunction on the moving barrier: upon hitting the barrier, the Gaussian becomes wider. When the rate of the spread is faster than the motion in free space, earlier arrival times (relative to free motion) will be observed. The dynamical interpretation is particularly clear by resorting to the path integral form of the time evolution operator $K(x, x', t)$. The Lagrangian with the potential (2) is quadratic and therefore $K(x, x', t)$ is given by

$$K(x, x', t - t_0) = \sqrt{\frac{i}{2\pi\hbar}\frac{\partial^2 S_{cl}}{\partial x \partial x'}} \exp \frac{i}{\hbar} S_{cl}(x, x', t - t_0), \tag{11}$$

where S_{cl} is the action for the classical paths going from x' to x in time $t - t_0$. Its explicit expression is cumbersome; it is obtained by employing a method similar to the one used for the time-dependent harmonic oscillator [14], namely by expressing the time integral of the Lagrangian as a quadratic form in x and x'. This is done here by employing the Ermakov system decomposition $q(t) = \sqrt{2I}\alpha(t) \sin\phi(t)$.

The important message encapsulated by the propagator expression (11) is that every point x' of the initial wavefunction $\psi(x', t_0)$ is carried to the point (x, t) by a classical trajectory: a single propagating wavepacket is built on the entire set of paths whose initial conditions lie within $\rho(x, t_0)$ (now regarded as a configuration space classical distribution). The corresponding family of trajectories for the arrival times shown in figure 2 is plotted in figure 3, along with the free trajectories for the same case. It can be seen that the barrier causes the impinging trajectories to accelerate or to turn back, depending on their initial positions. Whereas a potential barrier will typically prevent the particles from crossing, a rising barrier will transmit a kick and hence accelerate those trajectories that are found to the right of the barrier maximum while the barrier is rising. Earlier arrivals (relative to the no-barrier situation) are precisely produced by these paths that arrive at some point x_T far to the right of the barrier in a shorter time than in the free case.

Hence from a dynamical standpoint the arrival times can be understood from the behavior of classical distributions. However a wavepacket cannot be reduced to a classical statistical distribution: it is a quantum object that diffracts on the barrier. Therefore even though the underlying dynamics leading to early arrivals can be perfectly understood classically[5], some

[4] This definition emphasizes the detection time t_d in the barrier case relative to the velocity v_g of the maximum of the free Gaussian, keeping in mind that the maxima of the free and perturbed Gaussians move together up to times $t = t_B$.

[5] As mentioned above, this is due to the quadratic character of the potential. For non-quadratic potentials, the propagator is expanded beyond the first-order term (11), giving rise to a sum over paths including non-classical trajectories (diffractive or complex trajectories).

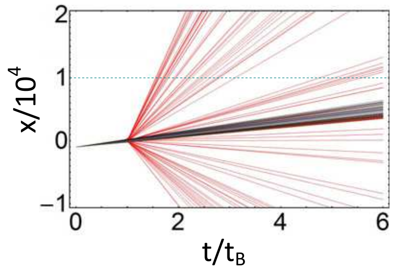

Figure 3. Trajectories $q(t)$ for the situation portrayed in figure 2 (black lines: free case; grey (red online) lines: time-dependent barrier with $k = 1/500$). The initial conditions for each trajectory are randomly chosen within the initial distribution $\rho(x, t_0)$, centered at $x = -1000$. The barrier potential, maximal at $t = t_B$, highly spreads the pencil of trajectories, resulting in the diffraction of the wavepacket.

effects can be specifically quantum (such as in entanglement generation, as described below in section 4). Note that the mechanism producing early arrivals in the present case is qualitatively different than the pulse reshaping involving evanescent waves in a dispersive medium giving rise to superluminal group velocities, see e.g. [15, 16]. Here, the diffraction on the barrier distorts essentially the tails of the wavepacket, but otherwise there is no other significant pulse reshaping.

4. Toward possible applications

We present in this section two illustrations implementing the early arrivals phenomenon. These illustrations, concerning a scheme for key generation and a procedure to speed-up entanglement between two qubits, may be considered as outlines needing further development in order to enable the closer examination of the actual realizability of the envisaged applications.

4.1. Key generation scheme using early arrivals

We have seen above that the transmission probability for the perturbed barrier exceeds that of the free case in a particular time interval. The detector during this time interval therefore records more number of particles than it would have in the free case. We have also provided an expression to measure how fast the information about perturbation of the wavepacket travels to the detector. In order for Alice and Bob to use this effect for a key generation, we now suppose that Alice and Bob perform the following experiment. Alice at the barrier receives single particles, one by one, prepared in the given Gaussian state. At first, she lets the particles move on without doing anything, and Bob at the detector, which is at a fixed location, detects the number of particles reaching the specified location for a particular chosen instant of time (position measurement). Then the chosen instant of time is varied, and the procedure is repeated for many such instants, thereby generating the curve $T_f(t)$. Alice then introduces the barrier perturbation by choosing a particular value for the barrier strength k, which she fixes to be the same for all particles. Bob again repeats the earlier procedure for all particles, one by one, and in a way similar to the above is able to generate the curve $T_k(t)$. Next, by comparing T_f with T_k, Bob is able to obtain t_c, t_d and hence, Δ_t, and then compute η using equations (8) and (9). The whole experiment is repeated by Alice and Bob many times for randomly chosen different values of the barrier strength k by Alice, leading to the inference in the above manner of the corresponding magnitude $\eta(k)$ by Bob. A particular functional relation between k and $\eta(k)$ (say, curve $1(c)$) is chosen as a key which is shared by Alice and Bob.

It needs to be mentioned that the above method of key generation is different from the standard quantum key distribution scheme in which one element of the key is generated by transmitting one particle. However, in order to determine to what extent such a scheme could be secure, a more detailed analysis would be required. In particular, note that in our scheme, a single value of η corresponding to the given value of k has to be obtained by sending particles one after the other, thereby generating an ensemble of particles representing the whole wavepacket. Hence, the question as to whether any intervention by an eavesdropper, such as performing a measurement, can be detected through distortion of the relation between k and v_I is non-trivial, and for example, would depend upon the choice of the subset of the ensemble selected for the purpose of eavesdropping. A quantitative comparison with security issues involved in usual quantum key distribution schemes (see, for example, [18]), would obviously be interesting, depending upon the extent to which our scheme can be comprehensively developed.

4.2. Effect on entanglement generation

The phenomenon inducing earlier arrival times can also be employed to speed-up the generation of entanglement between qubits mediated by a wavepacket. Note that scattering-based strategies aiming at entangling qubits have become increasingly popular in quantum information processing tasks [19]. Assume Alice, sitting at $x_A = q_0$ wants to send shared information using a single-particle wavepacket to Bob, placed to the right at $x_B = x_T > 0$ and to Charlie, sitting to Alice's left at $x_C = -x_T < 0$. Alice creates an initial state by superposing two Gaussians given by equation (1) with opposite mean momentum

$$\xi(x, t_0) = \left(\frac{8m}{\pi\alpha_0^2}\right)^{1/4} \mathrm{e}^{-m[x-q_0]^2/\alpha_0^2} \cos(p_0[x - q_0]/\hbar). \tag{12}$$

Bob and Charlie possess qubits, initially in the 'off' state $|0\rangle_B$ and $|0\rangle_C$. We assume an effective unitary interaction between the particle wavepacket and the qubits by which $|0\rangle_B$ is switched in a very short time (relative to the other timescales of the problem) into the 'on' state $|1\rangle_B$ provided $x > x_B$ and similarly $|0\rangle_C$ is switched into $|1\rangle_C$ for $x < x_C$; the backreaction on the wavepacket is supposed to be negligible. The initial total quantum state is a product state of the initial superposed Gaussians (12) and the qubits in the off-state:

$$|\Psi(t_0)\rangle = |\xi(t_0)\rangle|0\rangle_B|0\rangle_C. \tag{13}$$

The state subsequently evolves into the entangled wavepacket-qubits state

$$\begin{aligned}\langle x|\Psi(t)\rangle = \frac{1}{N^{1/2}}\xi(x, t)\big[&\theta(x_B - x)\theta(x - x_C)|0\rangle_B|0\rangle_C \\ &+ \theta(x - x_B)|1\rangle_B|0\rangle_C + \theta(x_C - x)|0\rangle_B|1\rangle_C\big],\end{aligned} \tag{14}$$

where $\xi(x, t)$ is the wavefunction evolved from (12), i.e. the sum of two wavepackets, one propagating toward Bob and the other toward Charlie; θ denotes the unit-step function and N is the normalization constant. Note that for small t, $\xi(x, t)$ is nonzero only in the interval $x_C < x < x_B$ and $|\Psi(t)\rangle$ remains a product state, whereas for large t, $\xi(x, t)$ has non-negligible amplitude for $x > x_B$ and $x < x_C$: the qubits are entangled and $N \approx \int_{x_B}^{+\infty} \mathrm{d}x\, |\psi(x, t)|^2 + \int_{-\infty}^{x_C} \mathrm{d}x\, |\psi(x, t)|^2$, that is, the sum of the transmission coefficients (6) at x_B and x_C.

The wavepacket can be seen as mediating the entanglement of Bob and Charlie's qubits. The corresponding entanglement rate can be quantified by computing the linear entropy

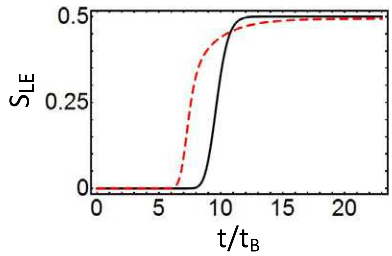

Figure 4. Entanglement buildup (quantified by the linear entropy $S_{\mathrm{LE}}(t)$) between Bob's and Charlie's qubits following the scheme described in section 4.2, for the free case (black solid curve) and in the presence of a transient barrier (grey (red online) dashed curve). The parameters taken are those corresponding to the transient barrier problem displayed in figure 2 with $k = 10^{-4}$, and $x_A = q_0, x_B = -x_C = 10^4$ (in atomic units).

associated with one or the other qubit's reduced density matrix. Starting from the total density matrix $\varrho(t) = |\Psi(t)\rangle\,\langle\Psi(t)|$, one computes first the reduced density matrix

$$\rho_q(t) = \int \mathrm{d}x \langle x|\Psi(t)\rangle\langle\Psi(t)|x\rangle \tag{15}$$

associated with the entangled qubits, depending again on the transmission coefficients (6) at x_B and x_C. Then by tracing over either Bob's or Charlie's qubit states, one obtains the density matrix $\rho(t)$ associated with a single qubit. The linear entropy S_{LE} is defined as

$$S_{\mathrm{LE}}(t) = 1 - Tr\,[\rho(t)]^2; \tag{16}$$

$S_{\mathrm{LE}}(t)$ vanishes for product states and here its highest value is $1/2$, reached when the qubits are maximally entangled.

Entanglement speed-up can be obtained if Alice combines the procedure we have just mentioned with a weak barrier centered at $x = 0$ that is raised and lowered for a short moment around $t = t_B$, that is, when the part of the wavepacket sent to Bob arrives at $x = 0$. Then as explained in section 2, the diffraction of the wavepacket on the barrier will result in parts of the wavepacket reaching Bob's qubit earlier than in the barrier-free case. The barrier must be weak in order to ensure no appreciable part of the wavepacket traveling toward Bob is reflected. Note that the appearance of a time-dependent barrier has no effect on the wavepacket traveling toward Charlie.

An example of entanglement speed-up is shown in figure 4, which compares the linear entropy $S_{\mathrm{LE}}(t)$ in the free case with the linear entropy computed with identical parameters but in the presence of a weak barrier. Significative entanglement is generated earlier than in the free case, although the maximally entangled state is obtained at longer times than in the free case. This is a straightforward consequence of the dynamical explanation given in section 3: a fraction of the wavepacket is accelerated by the rising barrier, allowing for an initial faster entanglement buildup, while a fraction of the wavepacket is decelerated by the barrier, so that the reduced linear entropy attains its maximum later than in the free case. The fact that entanglement—a specific quantum feature—reflects the underlying dynamics (even when the latter is purely classical) is a result that holds well beyond the specific example studied here and that has given rise to several investigations in the last decade including in scattering situations (see e.g. [20] and references therein).

5. Discussion and conclusion

In this work, we have analytically investigated the effect by which the transmission probability of a wavepacket impinging on a transient barrier exceeds that corresponding to free propagation for a certain interval of time. We have further introduced a scheme to quantify the magnitude of this effect attributable to the early arrivals of particles within a small duration of time. Although quantum mechanically this effect originates from the interaction of the Schrodinger wavefunction with a time-dependent barrier, its dynamical interpretation can be obtained, by using the semiclassical propagator, in terms of classical trajectories, with each trajectory carrying a part of the diffracted wave. Here, as in other cases in which the incident wave packet gets distorted on striking the barrier, the concept of signal velocity needs to be defined operationally, and it can exceed the group velocity of the wavepacket [15–17]. Accordingly, we have introduced a form of information velocity which measures the speed at which information about the barrier perturbation propagates across the wavepacket.

Furthermore, we have suggested the possibility of this effect being used for communication across the wavepacket. A preliminary idea of a simple scheme of information transfer has been outlined which is based upon the one-to-one correspondence between any particular value of the barrier strength chosen and the measured value of the magnitude of the early arrivals. In such a scheme, whose details require elaboration in a future work, the transfer of classical information using the quantum wavefunction is quite different from usual quantum key generation schemes. Finally, we have presented a simple scheme by which this transmission across a transient barrier can speed up the generation of entanglement. Further work involving the time-dependent reflection and transmission of wavepackets from various types of transient barriers is required for the purpose of experimental tests and applications of the effect demonstrated and analyzed in this paper.

Acknowledgments

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