Reply to "Comment on 'Relevance of Bell's theorem as a signature of nonlocality: Case of classical angular momentum distributions' "

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In this Reply, we show that the main argument of the criticism given in the preceding Comment is inconsistent with the assumption lying at the basis of the model introduced in the original paper [A. Matzkin, Phys. Rev. A **77**, 062110 (2008)]. We further clarify some issues concerning the interplay between noncommuting measurements and conservation laws in order to reply to the additional criticism contained in the comment.

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In a recent paper [1], I examined models involving classical angular momentum distributions in light of the Bell inequalities. Since it is well known that the existence of joint probability distributions lies at the origin of the inadequacy of Bell-type hidden variables models to account for quantum-mechanical expectation values, the idea was to come up with a classical model that would forbid joint distributions. Such a model was presented in Sec. IV of [1]. This model involves classical distributions and yields exactly the same expectation value that is obtained in the quantum Einstein-Podolsky-Rosen (EPR) –Bohm setting, known to violate the Bell inequalities. Section V of [1] discussed to what extent nonlocality needed to be *explicitly* involved in order to account for the classical results. Indeed, since the properties of the model arise by combining noncommutativity (precluding factorizability, see Sec. V B of [1]) and a conservation law (imposed by rotational invariance) in view of accounting for *correlated* outcomes, having recourse to nonlocality or asserting that a conservation law is all that is needed becomes a matter of taste—the central point being that without some sort of interparticle action, nonfactorizable models cannot account for *any* type of correlation between outcomes. Note that [1] did not deal at all with quantum mechanics.

In the preceding Comment [2], Tung appears to disagree with several statements made in [1]. The main point discussed by Tung is an attempt to show that the model of Sec. IV (that violates the Bell inequalities) is equivalent to the model of Sec. III C (that does not). This attempt is based on pinpointing the position of particle 2's angular momentum $\mathbf{J}_2$ to a given position on the sphere. The pinpointing process seems to be done by inferring the position of $\mathbf{J}_2$ from the known position of $\mathbf{J}_1$. Tung did not explain how this inference is made (in Ref. [2] of the Comment it is indicated that counterfactual reasoning plays a role).

Nevertheless, it is easy to establish that the pinpointing process is *inconsistent* with the model of Sec. IV. Actually the point was already made in Sec. IV A 1 of [1] where I wrote below Eq. (49) that "the main property of this particle-detector interaction based model is that the detected result does not depend on a phase-space point (or on a given individual position of the particle's angular momentum on the sphere)…." The proof was given right after. It was moreover repeatedly emphasized that this main property—the inconsistency of the model with the existence of elementary probability functions depending on an individual position of $\mathbf{J}$—holds relative to a single particle and is thus not a consequence of the correlation in a two-particle system. A property that does not hold in a single-particle system cannot be expected to be valid in a two-particle system.

We will not repeat the proof given in Eqs. (50) and (51) of [1] here. The important point to bear in mind is that the defining assumption of the model is, for a single particle,

$$\langle R_b\rangle_{\rho_{a+}} = \sum_{k=-1/2}^{1/2} kP(R_b=k,\rho_{a+}) = \langle J_b\rangle_{\rho_{a+}} = \frac{1}{2}\cos(\theta_b-\theta_a), \tag{1}$$

where $R_b=\pm\frac{1}{2}$ denotes the outcome for measurements along the b axis. Equation (1) means that the average obtained over the discrete measurement outcomes matches the average angular momentum projection J_b over the distribution ρ_{a+} (a uniform distribution localized on the hemispherical surface Σ_{a+}). Equation (1) along with normalization of the $P(R_b=k,\rho_{a+})$ is sufficient to impose the value of the probabilities

$$P\left(R_b=\pm\frac{1}{2},\rho_{a+}\right) = \frac{1\pm\cos(\theta_b-\theta_a)}{2}. \tag{2}$$

On the other hand, assume the existence of elementary probability functions $p(R_b=\pm\frac{1}{2},\mathbf{J})$ depending only on $\mathbf{J}$ such that

$$P\left(R_b=\pm\frac{1}{2},\rho_{a+}\right) = \int p\left(R_b=\pm\frac{1}{2},\mathbf{J}\right)\rho_{a+}(\mathbf{J})d\mathbf{J}. \tag{3}$$

Then because we must have $p(R_b=\pm\frac{1}{2},\mathbf{J})=1$ for any $\mathbf{J}\in\rho_{b+}$ and $p(R_b=\pm\frac{1}{2},\mathbf{J})=0$ for any $\mathbf{J}\in\rho_{b-}$ [in order to comply with Eq. (2) when $b=a$], and because the distributions are uniform, Eq. (3) becomes

$$\int p\left(R_b=\pm\frac{1}{2},\mathbf{J}\right)\rho_{a+}(\mathbf{J})d\mathbf{J} = \int_{\Sigma_{a+}\cap\Sigma_{b+}} d\mathbf{J} = 1-\frac{\theta_b-\theta_a}{\pi}. \tag{4}$$

This result obviously contradicts Eq. (2) thereby showing that even in the single-particle case, assuming probability functions $p(R_b=\pm\frac{1}{2},\mathbf{J})$ is inconsistent with the basic assumption (1) of the model, except if the elementary probabilities further depend on the distributions (see Appendix B of [3], where a step-by-step derivation is given). Thus it makes no sense to pinpoint an individual position of the angular momentum on the sphere; Tung's assertion by which the model of Sec. IV "*reveals itself to be just a disguise*" of the model of Sec. III is refuted. From a physical standpoint there are many classical situations leading to the nonexistence of elementary probability functions. For example, imagine a single particle having a stochastic motion uniform on Σ_{a+} characterized by $J_a > 0$ and $\langle J_a \rangle = 1/2$. If the time scale of this motion is much shorter than the time scale of a measurement, R_a will appear as the time average of the particle's angular momentum projection on the a axis; see Eq. (53) of [1]. If R_b is measured instead, by the time the measurement is over the stochastic motion will have changed—precisely because of the measurement interaction—from being stochastic over Σ_{a+} previous to the measurement to being stochastic over either Σ_{b+} or Σ_{b-} (corresponding to the outcomes $R_b = +1/2$ or $-1/2$) at the end. It is not possible to pinpoint the angular momentum to a fixed position $\mathbf{J}$ for which the projection J_q would be well defined along any number of arbitrary axes q: a measurement can only change the hemisphere over which the stochastic motion takes place. The pinpointing process simply does not make sense within this model. Again, this has nothing to do with the issue of locality, but is instead related to the noncommutativity of these classical measurements which does not allow one to obtain jointly outcomes R_q resulting from measurements along different axes q [since a measurement modifies the premeasurement situation and leads to a post measurement situation incompatible with that arising from a measurement made along a different axis (see Sec. II A)].

Finally, I respond to the other criticism made in [2] formulated in very brief remarks. First, Tung asserted that the model involves non-pre-existing conserved quantities that do not affect one another. This is wrong: as recalled above they do affect one another, so that $R_{1q} = -R_{2q}$ holds for any axis q, via a conservation law (which can be postulated to hold as such or be transmitted through a field or produced by a nonlocal mechanism). Then Tung stated that the present model cannot be taken as a hidden-variable theory accounting for the quantum correlations. I agree: [1] is not concerned by quantum mechanics but by Bell's theorem—which is a mathematical result unrelated to any specific physical theory—and its possible meaning when dealing with certain types of distributions in classical mechanics. It is also asserted in [2] that a field transport of the angular momentum would imply infinite velocities in order to explain the correlations of simultaneous quantum measurements. Again, [1] did not discuss the quantum-mechanical situation, though it may be remarked that from an experimental perspective "simultaneous" (whatever the specific frame of reference) means imposing time constraints on models providing for some type of communication between the particles; in this particular context Leggett [4] recently argued that present experiments do not rule out wave-function collapse theories accounting for quantum correlations.

To conclude, the mistake made in [2] is to attribute to the model of Sec. IV properties specific of Bell-type models. The important underlying question is whether factorizability—which implies commutativity—is necessary in order to implement locality. If locality is defined so as to preclude any type of action (through communication or "influence" as could be the case for conservation laws) between the particles and/or measurement apparata, then it appears that positively defined classical distributions of particles will not be able to match the correlations obtained in the EPR quantum type of experiments. Bell-type factorizable models will obey the relevant Bell inequality; noncommutative models (such as the one given in Sec. IV of [1]) will violate trivially the Bell inequality, i.e., with the bound being 4 as expected for independent noncorrelated events [5].

[1] A. Matzkin, Phys. Rev. A **77**, 062110 (2008).
[2] Ten Yong Tung, preceding Comment, Phys. Rev. A **79**, 046101 (2009).
[3] A. Matzkin, J. Phys. A **41**, 085303 (2008).
[4] A. J. Leggett, J. Phys. A **40**, 3141 (2007).
[5] Since noncommutative models are not compatible with the existence of joint distributions, the outcomes can be regarded as being independent in the absence of correlations.